

IN MEMORY OF TIBOR GALLAI  
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Ten years ago, when Tibor Gallai turned 70, I wrote a paper in this journal [1] of my reminiscences and remarks of our joint work with Gallai. Now that he had died, I write again, in his memory.

There, in [1] I concentrated on our joint work in analysis (theory of polynomials), and on graph theory. There is not much reason to repeat here what can be found there, except perhaps one thing. I stated in [1] that several of his results were later independently discovered and published by others. However, since Gallai was very modest, I had to promise to him (long ago) never to state in his lifetime these facts.

Unfortunately, now I can return to this question. I will mention only one case, the most important one.

In 1947 Gallai and Milgram discovered the result which is now known as the Theorem of Dilworth. The result was completely proved by Gallai and Milgram. Gallai wrote up the paper in German, but Milgram preferred to publish it in English. Unfortunately, Gallai did not write easily in English and Milgram, a very good topologist, did not realize the importance of this result. So Milgram delayed writing it up. Then early in 1950, Dilworth discovered his theorem. My mother wrote me: “Turán just visited me with the *Annals of Mathematics* and very excitedly showed me the paper and said: This is now Dilworth’s Theorem and ever more shall be so.” This was the only case when Gallai was sorry that he did not publish the result in time, he told me many years later. “I should just have published our joint paper in German.”

There is also a written testimony from 1947 indicating that the result of Gallai–Milgram existed: see my paper [2]. At the end of the paper I state the following result.

**Theorem.** *Let there be given a graph of  $(k-1)(\ell-1)+1$  vertices. Then either  $G$  contains a complete graph of order  $k$  or  $G'$  contains a directed path of  $\ell$  vertices for every orientation of the edges of  $G'$  in which there are no directed closed paths. ( $G'$  is the complementary graph of  $G$ ).*

Then I state: *Recently very much more general theorems have been proved by Gallai and Milgram.* One of these very much more general theorems was the Theorem of Dilworth.

As I stated there, Tibor Gallai was one of my oldest friends.

The last years of Gallai were very unhappy. In 1973 his wife Ibolya Dusák, also a mathematician, died after a long illness. The illness of Ibi Dusák made Gallai’s last years very difficult and prevented him from active research. Still, my last sentence from that paper [1] was “We all hope that he will long continue to be active mathematically and I certainly hope that our last joint paper has not yet been written.” This wish at least has been fulfilled. A paper of Gallai, Tuza and myself is in print in *Discrete Mathematics* [3]. This paper contains many interesting results and unsolved problems.

A few years before his death he finally accepted the degree of Doctor of the Academy and two years ago, much against his will, he was even granted the membership in the Academy.

Mike Plummer wrote about Gallai that he was “one of the unsung heroes of graph theory”.

Hilbert, in his obituary of Minkowski in the *Mathematische Annalen* (1909) wrote that “I can only be grateful that I had a friend and co-worker for such a long time”. This is what I have to say about Gallai and that “May his theorems live forever.”

### References

- [1] P. ERDŐS: Personal reminiscences and remarks on the mathematical work of Tibor Gallai, *Combinatorica*, **2** (1982), 207–212.
- [2] P. ERDŐS: Some Remarks in the Theory of Graphs, *Bulletin of the American Mathematical Society* (1947), 292–294.
- [3] P. ERDŐS, T. GALLAI, and Zs. TUZA: Covering the cliques of a graph with vertices, *Discrete Mathematics*, (in print).